**Biler, Piotr**

**Global solutions to some parabolic-elliptic systems of chemotaxis.** (English) Zbl 0941.35009

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Global time solvability is studied for the parabolic-elliptic system: $u_t = \nabla \cdot (\nabla u - u\nabla\varphi(v))$, in $\Omega \times \mathbb{R}^+$ with Neumann boundary conditions both for $u$ and $v$. The present author generalizes results from [*T. Nagai* and *T. Senba*, Adv. Math. Sci. Appl. 8, No. 1, 145-156 (1998; Zbl 0902.35010)] for radial domains separately in three directions, namely more general "sensitivity" functions $\varphi$, nonradial domains and with an extra diffusion term for $v$. The main settings are the following. 1) He treats $\varphi$ for the radial case with $N = 2$ under the conditions $0 < \varphi'(v) \to 0$ for $v \to \infty$ and $v\varphi'(v)$ increasing. 2) For $\varphi(v) = \chi \log v$ with $0 < \chi < 2/N$ $(N \geq 2)$, and $u(\cdot, 0) \in L^q(\Omega)$ with $q > N/2$ he considers the case of general domains. 3) Assuming $N = 2$, $\varphi(v) = \chi \log v$ with $0 < \chi < 1$ and $\nabla v(\cdot, 0) \in L^2(\Omega)$, the second equation is replaced by $\varepsilon v_t - \Delta v + v = u$. In all three cases he proves that the solution for the initial value problem exists globally in time. The main step in the proofs is to show an a priori bound for $u(\cdot, t)$ which uses some delicate nonlinear analysis. The system has been considered by the same author in earlier papers. The present results are rather sharp; slightly different sensitivity functions have displayed blow-up in finite time.

Reviewer: G.H.Sweers (Delft)

**MSC:**

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|---|---|
| 35B40 | Asymptotic behavior of solutions to PDEs |
| 92C45 | Kinetics in biochemical problems (pharmacokinetics, enzyme kinetics, etc.) |
| 35K60 | Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations |



**Keywords:**

sensitivity functions