

McMullen, C. T.

Lipschitz maps and nets in Euclidean space. (English) Zbl 0941.37030
Geom. Funct. Anal. 8, No. 2, 304-314 (1998).

This paper deals with the following three questions:

1. Is there a bi-Lipschitz homeomorphism $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that the Jacobian determinant $\det D\varphi = f$, where f is a given real-valued function $f \in L^\infty(\mathbb{R}^n)$ with $\inf f(x) > 0$?
2. Is there a Lipschitz or quasiconformal vector field with $\operatorname{div} v = f$, where f is a given $f \in L^\infty(\mathbb{R}^n)$?
3. Given a separated net $Y \subset \mathbb{R}^n$, is there a bi-Lipschitz map $\varphi : Y \rightarrow \mathbb{Z}^n$?

Note that in the case of $n = 1$ all three questions have a positive answer. The author shows that for $n > 1$ the answer to all three questions is negative. He also proves that all three questions have positive solutions if the Lipschitz condition is replaced by a Hölder condition.

Reviewer: [Messoud Efendiev \(Berlin\)](#)

MSC:

- 37F30** Quasiconformal methods and Teichmüller theory, etc. (dynamical systems) (MSC2010)
- 37F05** Dynamical systems involving relations and correspondences in one complex variable

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Keywords:

[bi-Lipschitz homeomorphism](#); [Jacobian determinant](#); [quasiconformal vector field](#); [bi-Lipschitz map](#)

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