Sabitov, I. Kh.

A generalized Heron-Tartaglia formula and some of its consequences. (English. Russian original) Zbl 0941.52020

Sb. Math. 189, No. 10, 1533-1561 (1998); translation from Mat. Sb. 189, No. 10, 105-134 (1998).

Recall Tartaglia's formula for the volume of a tetrahedron which can be considered as a natural generalization of the Heron area formula for a triangle. Let T be a tetrahedron with edge lengths $l_1, l_2, l_3, l_4, l_5, l_6$, then its volume V is given by the formula

$$V^2 = \frac{1}{144}\big(l_1^2 l_5^2 (l_2^2 + l_3^2 + l_4^2 + l_6^2 - l_1^2 - l_5^2) + l_2^2 l_6^2 (l_1^2 + l_3^2 + l_4^2 + l_5^2 - l_2^2 - l_6^2)$$

$$+ l_3^2 l_4^2 (l_1^2 + l_2^2 + l_5^2 + l_6^2 - l_3^2 - l_4^2) - l_1^2 l_2^2 l_4^2 - l_2^2 l_3^2 l_5^2 - l_1^2 l_3^2 l_6^2 - l_4^2 l_5^2 l_6^2\big).$$

The aim of the paper is to obtain a similar result for arbitrary polyhedron with triangular faces. More precisely, it is shown that for triangular faced polyhedron with a given combinatorial structure K and a given collection (l) of edge lengths there exists an even polynomial P depending on K and (l) and not on the concrete configuration of polyhedron itself such that the volume of the polyhedron is a root of P. The coefficients of P are themselves polynomial in the squares of the edge lengths of the polyhedron. For some partial cases, for example for octahedron with equal lengths of opposite edges an explicit formula for P is given.

The present proof of this theorem was given first in the author's doctoral dissertation "Isometric maps, flexes, and volumes in the metric theory of surfaces and polyhedra", Moscow State University, Moscow (1997), (Russian). The following consequence of the theorem gives a positive answer on the R. Connelly question posed at the 1978 International Congress of Mathematicians in Helsinki. ['The volume of flexible polyhedron stays constant during a flex'].

Reviewer: A.D.Mednykh (Novosibirsk)

MSC:

52C25	Rigidity and flexibility of structures (aspects of discrete geometry)
51M25	Length, area and volume in real or complex geometry
52B10	Three-dimensional polytopes

Keywords:

flexible polyhedra; volume; combinatorial structure

Full Text: DOI