

**Montagna, Franco**

**An algebraic approach to propositional fuzzy logic.** (English) Zbl 0942.06006  
*J. Logic Lang. Inf.* 9, No. 1, 91-124 (2000).

The problem of adding a product connective to MV-algebras has attracted increasing attention in recent years. One source of interest comes from MV-algebraic probability theory where, generalizing Carathéodory's Boolean algebraic probability, one tries to develop non-commutative probabilistic notions using the equivalence between MV-algebras and certain limits of finite-dimensional AF  $C^*$ -algebras. For background see the monograph by *R. L. O. Cignoli, I. M. L. D'Ottaviano* and *D. Mundici* [Algebraic foundations of many-valued reasoning (Trends in Logic – Studia Logica Library 7, Kluwer Academic Publishers, Dordrecht) (2000; [Zbl 0937.06009](#))], as well as the monograph by *B. Riečan* and *T. Neubrunn* [Integral, measure, and ordering (Kluwer, Dordrecht) (1997; [Zbl 0916.28001](#))].

Another source of interest comes from *P. Hájek's* work on the logic of t-norms, as expounded in his monograph: *Metamathematics of fuzzy logic* (Kluwer, Dordrecht) (1998; [Zbl 0937.03030](#)), showing the central role of the Łukasiewicz connectives, together with product.

A third motivation is given by the general program of giving “infinite-valued” definitions of such notions as set, Cartesian product, equality, eventually yielding a complete “first-order” infinite-valued logic.

In the present paper the author investigates various classes of algebras containing the Łukasiewicz operations, together with product. For some of these classes the author is able to extend the categorical equivalence between MV-algebras and Abelian lattice-ordered groups with strong unit proved in the present reviewer's paper “Interpretation of AF  $C^*$ -algebras in Łukasiewicz sentential calculus” [*J. Funct. Anal.* 65, 15-63 (1986; [Zbl 0597.46059](#))]. Thus, for instance, the author proves that a certain category of MV-algebras with product where one can define “one half” is equivalent to the class of f-semifields. The problems considered in this paper are naturally related to other parts of mathematics, such as real closed fields, and a variant of the Birkhoff-Pierce problem concerning free algebras in the variety generated by the unit real interval equipped with truncated addition, truncated subtraction, and multiplication.

Reviewer: [D.Mundici \(Milano\)](#)

**MSC:**

[06D35](#) MV-algebras  
[03B50](#) Many-valued logic  
[03B52](#) Fuzzy logic; logic of vagueness

Cited in **2** Reviews  
Cited in **43** Documents

**Keywords:**

[Łukasiewicz logic](#); [product connective](#); [MV-algebras with product](#)

**Full Text:** [DOI](#)