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An extension of the notion of universal Taylor series. (English) Zbl 0942.30003

Papamichael, N. (ed.) et al., Proceedings of the 3rd CMFT conference on computational methods and function theory 1997, Nicosia, Cyprus, October 13-17, 1997. Singapore: World Scientific. Ser. Approx. Decompos. 11, 421-430 (1999).

Let $\Omega \subset \mathbb{C}$ be a simply connected domain, $\Omega \neq \mathbb{C}$, and denote by $H(\Omega)$ the space of holomorphic functions in Ω with the usual topology of locally uniform convergence. For $f \in H(\Omega)$, $\zeta \in \Omega$ and a non-negative integer N let $S_N(f, \zeta, \cdot)$ denote the N-th partial sum of the Taylor expansion of f with centre ζ. Then the main result of the article under review states that there is a universal function $f \in H(\Omega)$ in the following sense: For every compact set $K \subset \mathbb{C}$, $K \cap \Omega = \emptyset$ with $\mathbb{C} \setminus K$ connected and every function $\varphi\colon K \to \mathbb{C}$ continuous on K and holomorphic in the interior K° of K, there is a strictly increasing sequence (λ_n) of non-negative integers such that for every compact set $L \subset \Omega$ there holds

$$\sup_{\zeta \in L} \sup_{z \in K} |S_{\lambda_n}(f, \zeta, z) - \varphi(z)| \to 0, \quad \text{as } n \to \infty.$$

Moreover, the set $U(\Omega)$ of all these universal functions is a countable intersection of dense open subsets of $H(\Omega)$. In particular, $U(\Omega)$ is of the second Baire category in $H(\Omega)$. The main tools of the proof are Mergelyan's approximation theorem and Baire's category theorem.

Universal holomorphic functions in various senses have been studied by several authors. A short survey is given in the text. The new element in this article is that K may meet the boundary of Ω.

The author also gives some interesting consequences of the main result. For example, any $f \in H(\Omega)$ may be expressed as the sum of two universal functions in $U(\Omega)$, and neither $f \in U(\Omega)$ is rational nor does it extend continuously on $\overline{\Omega}$.

For the entire collection see [Zbl 0921.00019].

Reviewer: Rainer Brück (Giessen)

MSC:

30B30 Boundary behavior of power series in one complex variable; overconvergence

30E10 Approximation in the complex plane

Keywords:

universal holomorphic functions; approximation in the complex domain