Detchenya, L. V.; **Sadovskij, A. P.**

Orbital normal forms of two-dimensional analytic systems with zero linear part and nonzero quadratic part. (English. Russian original) Zbl 0944.34031

Differ. Equations 35, No. 1, 50-56 (1999); translation from Differ. Uravn. 35, No. 1, 51-57 (1999).

The authors classify all possible local phase portraits of the planar systems

$$\begin{aligned} dx/dt &= A_1x^2 + A_2xy + A_3y^2 + X(x,y), \\ dy/dt &= -B_1x^2 - B_2xy - B_3y^2 - Y(x,y), \end{aligned} \qquad (*)$$

where X, Y are real-analytic in a neighborhood of the origin whose Taylor series start with third-order terms. Using affine transformations, *K. S. Sibirskij* had previously given a classification of $(*)$ with respect to the quadratic terms only [Differ. Equations 22, 669-674 (1986); translation from Differ. Uravn. 22, No. 6, 954-961 (1986; Zbl 0618.34027) and Introduction to the algebraic theory of invariants of differential equations. Transl. from the Russian. Nonlinear Science: Theory and Applications. Manchester etc.: Manchester University Press (1988; Zbl 0691.34031)]. He thus obtained nine types of systems $(*)$.

The present authors apply the formal transformations

$$x' = x + \sum_{i+j\geq 2} \alpha_{ij}x^iy^j, \quad y' = y + \sum_{i+j\geq 2} \beta_{ij}x^iy^j,$$

$$dt' = \left(1 + \sum_{i+j\geq 1} \gamma_{ij}x^iy^j\right)dt$$

to Sibirskij's normal forms to classify the terms of order ≥ 3 as well. In this way, they obtain forty-nine formal "orbital normal forms" of $(*)$. For example, Sibirskij's normal form

$$dx/dt = x^2 + p(x,y), \quad dy/dt = xy - q(x,y),$$

is reduced to the system

$$dx/dt = x^2 + \sum_{k\geq 3} C_ky^k, \quad dy/dt = xy.$$

(Here, x', y', t' are replaced by x, y, t again.) However, some of Sibirskij's normal forms are transformed into a considerable number of different orbital normal forms. The local phase portraits of all these orbital normal forms and, hence, all possible local phase portraits of system $(*)$, can now be constructed.

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MSC:

34C20 Transformation and reduction of ordinary differential equations and systems, normal forms

34C07 Theory of limit cycles of polynomial and analytic vector fields (existence, uniqueness, bounds, Hilbert's 16th problem and ramifications) for ordinary differential equations

37C10 Dynamics induced by flows and semiflows

Keywords:

planar vector field; normal form; phase portrait