Mumford, David
The red book of varieties and schemes. Includes the Michigan lectures (1974) on “Curves and their Jacobians”. 2nd, expanded ed. with contributions by Enrico Arbarello. (English) Zbl 0945.14001

D. Mumford’s famous Harvard lecture notes “Introduction to algebraic geometry: Preliminary version of the first three chapters” appeared in the mid 60’s. These notes were a typed-up version of his class room notes, distributed in mimeographed form by the Mathematics Department at Harvard University and were bound in red. At that time, Mumford’s red booklet was actually the first available down-to-earth introduction to the modern approach to algebraic geometry via algebraic varieties, schemes, sheaves, and morphisms. In fact, the well-known standard textbooks on modern algebraic geometry, which have been used over the past decades, have appeared only years or decades later. Due to these circumstances, and due to the excellence of Mumford’s introductory text to varieties and schemes, these actually preliminary notes gained a tremendous popularity within the international mathematical community. The reason for this great, now seemingly “everlasting” popularity of Mumford’s (preliminary) notes is based on the fact that they were designed to just provide a very first, but rigorous and reasonably deep-going introduction (and invitation) to the basic concepts of modern algebraic geometry, together with a maximum of motivations and illustrations. – This quick path to the foundations of modern algebraic geometry, in its originally preliminary setting, has proved to be the greatest advantage of Mumford’s red booklet, over all the years. These notes cannot replace the study of a comprehensive textbook on the subject, and they were never supposed to do so, but they still represent the possibly best first step to the study of any advanced and complete textbook on modern algebraic geometry. David Mumford himself had intended to expand his “preliminary notes on the first three chapters” to a comprehensive textbook by including much more topics, a project that finally never became reality, but his red booklet was always there and maintained its unique character within the now steadily growing textbook literature on the subject.

For this reason, a reprinted version of Mumford’s famous notes was published by Springer Verlag in 1988, this time under the honorary title “The red book of varieties and schemes” [Lect. Notes Math. 1358 (1988; Zbl 0658.14001)]. Now, after more than ten years, this edition had been sold out again. At about the same time, the supply of D. Mumford’s other famous lecture notes “Curves and their Jacobians” (published by the University of Michigan Press in 1975; Zbl 0316.14010) was also exhausted. As these notes had the same purpose as the “red book”, namely to provide an introductory, panoramic view of a certain topic in modern algebraic geometry, David Mumford and Springer Verlag decided to reprint both sets of notes together. – The book under review is now the second, expanded edition of D. Mumford’s “The red book of varieties and schemes”, with the original “Red book” as part one and the reprint of “Curves and their Jacobians” as part two, where both parts appear in their well-established original form.

“Curves and their Jacobians” is a slightly expanded version of a series of four lectures delivered by the author in 1974 at the University of Michigan, Ann Arbor. The aim of these lectures was to introduce people in the mathematical community at large to the beautiful topic of complex algebraic curves, their Jacobians, their moduli spaces, and to the classical Schottky problem at its contemporary state of art. In those lectures, D. Mumford gave a “bird’s eye view” (as he called it) of the following four central topics in the theory of complex algebraic curves (or compact Riemann surfaces):

Lecture I: What are curves and how explicitly can we describe them?
Lecture II: The moduli space of curves: Definition, coordinatization, and some properties;
Lecture III: How Jacobians and theta functions arise;
Lecture IV: The Torelli theorem and the Schottky problem.

This excellent, brilliant survey on complex algebraic curves and their moduli provided, among other inspiring expositions, the first systematic discussion of the various approaches to tackle the long-standing Riemann-Schottky problem of characterizing the Jacobians of curves among the principally polarized abelian varieties. These approaches were probably known by only a small number of specialists working in that area, and the Schottky problem was still an open question in the 70’s. An explosion of activity
in finally solving the 100-years-old Schottky problem broke loose in the early 80’s, shortly after the appearance of Mumford’s published notes “Curves and their Jacobians”, and ended up with several analytic solutions to the Schottky problem, which were contributed by various authors independently.

This is to say that Mumford’s survey notes certainly had some mobilizing impact on this development, at least so by their inimitably inspiring, clarifying, systematizing and trend-setting character, and this is what makes them another “classic evergreen” in the literature of algebraic geometry. The reprint of “Curves and their Jacobians” has been enhanced by an updating appendix “Survey of work on the Schottky problem up to 1996”, written by E. Arbarello and reflecting briefly the great progress that has been achieved, over the past twenty years, in solving the Schottky problem. This appendix of four pages is essentially a quick guide through the vast recent work on the Schottky problem, the bibliography of which (between 1975 and 1996) is added, also by E. Arbarello, under the title “Supplementary bibliography on the Schottky problem” at the end of the book.

Altogether, it is highly gratifying to see that D. Mumford’s two classics are still kept in print, very much so to the indispensable benefit of the younger, and of the coming generations of algebraic geometers.

Reviewer: Werner Kleinert (Berlin)

MSC:

14A10 Varieties and morphisms
14H42 Theta functions and curves; Schottky problem
14K25 Theta functions and abelian varieties
14-02 Research exposition (monographs, survey articles) pertaining to algebraic geometry
14-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to algebraic geometry
14A15 Schemes and morphisms

Keywords:

theta functions; Prym varieties; abelian varieties; red book of varieties and schemes; varieties; schemes; Jacobians; Schottky problem

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