This interesting paper deals with the spatially homogeneous Boltzmann equation
\[ \frac{\partial f}{\partial t}(t, v) = Q(f, f)(t, v), \quad (t, v) \in (0, +\infty) \times \mathbb{R}^3; \quad f(0, v) = f_0(v), \quad v \in \mathbb{R}^3, \]
where \( f(t, v) \) is a nonnegative function, which describes the time evolution of the distribution of particles, which move with velocity \( v \), and \( Q(f, f)(t, v) \) is the collision operator. The main result is that for any initial data in \( L^1_{\text{loc}}(\mathbb{R}^3) \) with finite mass and energy, there exists a unique solution \( f \in C([0, +\infty); L^1_{\text{loc}}(\mathbb{R}^3)) \) for which the same two quantities are conserved. Here \( L^1_{\text{loc}}(\mathbb{R}^3) \) denotes the space of all functions \( f \) such that \( \|f\|_{1,s} = \int_{\mathbb{R}^3} f(v)(1+|v|^2)dv \) is bounded. Another interesting statement is that any solution satisfying certain bounds on moments of order \( s \) must necessarily have bounded energy. A time discretization of the Boltzmann equation with \( f_0 \in L^1_s(\mathbb{R}^3) \ (s \geq 2) \) is considered as well. Some estimates of the rate of convergence for the explicit and implicit Euler schemes are given. Two auxiliary results are of independent interest: a sharpened form of the Povzner inequality, and a regularity result for an iterated gain term.

Reviewer: Dimitar Kolev (Sofia)

MSC:
- 35Q35 PDEs in connection with fluid mechanics
- 82C40 Kinetic theory of gases in time-dependent statistical mechanics
- 76P05 Rarefied gas flows, Boltzmann equation in fluid mechanics

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