Fraenkel, L. E.
An introduction to maximum principles and symmetry in elliptic problems. (English)
Zbl 0947.35002
Cambridge Tracts in Mathematics. 128. Cambridge: Cambridge University Press. x, 340 p. (2000).

The aim of the book is to present the basic theory of the symmetry of solutions of second order elliptic partial differential equations by means of the maximum principle. In Chapter 2, several kinds of maximum principles are included for linear second order operators. In Chapters 3 and 4, results on symmetry of solutions for a nonlinear Poisson equation in symmetric domains are discussed by using the well known method of moving plane introduced by Alexandrov (1956) and developed by Serrin (1971) and by *B. Gidas, W.-M. Ni* and *L. Nirenberg* in the celebrated paper "Symmetry and related properties via the maximum principle" [Commun. Math. Phys. 68, 209-243 (1979; Zbl 0425.35020)]. In Chapter 5, monotonicity of positive solutions in bounded sets are discussed.

The second part of the book contains five Appendices. Appendix A concerns Newtonian potential theory in $\mathbb{R}^3$. In Appendix B, general facts about harmonic functions and the Poisson equation are presented. Appendix D is devoted to the discussion of the Divergence Theorem under different conditions on the field as well as on the geometry of the boundary of the domain.

Each Chapter contains several exercises with suitable hints for the solution: most of them illustrate clearly some aspects of the theorems proved earlier.

The book is adequate for a graduate program and for any researcher who is interested in qualitative properties of the solutions of equations.

Reviewer: G.Porru (Cagliari)

MSC:

35-02	Research exposition (monographs, survey articles) pertaining to partial differential equations
35B50	Maximum principles in context of PDEs
35Jxx	Elliptic equations and elliptic systems
35A30	Geometric theory, characteristics, transformations in context of PDEs

Cited in **2** Reviews
Cited in **121** Documents

Keywords:

maximum principles; moving plane; symmetry; second order elliptic equations; nonlinear Poisson equation; monotonicity