Summary: Let $G = (V, E)$ be a capacitated directed graph with a source $s$ and $k$ terminals $t_i$ with demands $d_i$, $1 \leq i \leq k$. We would like to concurrently route every demand on a single path from $s$ to the corresponding terminal without violating the capacities. There are several interesting and important variations of this unsplittable flow problem.

If the necessary cut condition is satisfied, we show how to compute an unsplittable flow satisfying the demands such that the total flow through any edge exceeds its capacity by at most the maximum demand. For graphs in which all capacities are at least the maximum demand, we therefore obtain an unsplittable flow with congestion at most 2, and this result is best possible. Furthermore, we show that all demands can be routed unsplittably in 5 rounds, i.e., all demands can be collectively satisfied by the union of 5 unsplittable flows. Finally, we show that 22.6% of the total demand can be satisfied unsplittably.

These result are extended to the case when the cut condition is not necessarily satisfied. We derive a 2-approximation algorithm for congestion, a 5-approximation algorithm for the number of rounds and a $4.43 = 1/0.226$-approximation algorithm for the maximum routeable demand.

MSC:

- 90B10 Deterministic network models in operations research
- 90C35 Programming involving graphs or networks
- 05C38 Paths and cycles

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- unsplittable flow problem

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