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A perturbed integral geometry problem in a three-dimensional space. (English. Russian original) Zbl 0952.45001

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The article contains uniqueness theorems for the following integral equation in a function $u(\xi)$:

$$\int_{p(x)} u(\xi)d\bar{\xi} + \int_{P(x)} W(x,\xi)u(\xi)d\xi = f(x), \tag{1}$$

where $W(x,\xi)$ is a given weight function, $x \in \mathbb{R}^3$, $\xi \in \mathbb{R}^3$, $\overline{\xi} = (\xi_1, \xi_2)$, $p(x)$ is the projection of the paraboloid $\mathcal{P}(x) = \{\xi : x_3 - \xi_3 = |\overline{x} - \overline{\xi}|^2,\ 0 \leq \xi_3 \leq h\}$ onto the plane $x_3 = 0$, $\overline{x} = (x_1, x_2)$, $0 < h < \infty$. It is supposed that the vertices x of the paraboloids $\mathcal{P}(x)$ lie in the layer $S = \{x \in \mathbb{R}^3_+ : 0 < x_3 < h\}$, $\mathbb{R}^3_+ = \{x = (x_1, x_2, x_3) : x_3 \geq 0\}$. In (1), $P(x)$ is the part of the layer $\overline{S}$ which is bounded by the surface of $\mathcal{P}(x)$ and the plane $x_3 = 0$, where $\overline{S} = \{x \in \mathbb{R}^3_+ : 0 \leq x_3 \leq h\}$. Let $\mathcal{A} = \{x \in \mathbb{R}^3_+ : |x_k| \leq a_k\ (k = 1, 2)$, $a_k < \infty$, $0 < x_3 < h\}$.

The following theorem is the main result: A solution to (1) in the class $C^3_0(\mathcal{A})$ is unique if (a) the function $f(x)$ is known for all $x \in \overline{S}$ and (b) the weight function W has all continuous derivatives up to the second order and vanishes together with its derivatives on the surface of the paraboloid $\mathcal{P}(x)$.

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53C65 Integral geometry

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