Van de Geer, Sara A.

Applications of empirical process theory. (English) Zbl 0953.62049

Cambridge Series in Statistical and Probabilistic Mathematics 6. Cambridge: Cambridge University Press. xii, 286 p. (2000).

As stated in the preface, "this book is an extended version of a set of lecture notes ... for the ... course 'Applications of Empirical Process Theory', which ... was intended for students with a master of sciences in mathematics or statistics." Accordingly, the book provides an introduction into basic notions and results of modern empirical process theory and some of its applications for readers without previous knowledge in these areas. After a warming up in Chapter 1, Introduction, the necessary material on empirical processes is presented in the five chapters: 2. Notation and Definitions, 3. Uniform Laws of Large Numbers, 5. Increments of Empirical Processes, 6. Central Limit Theorems, and 8. The Non-I.I.D. Case. Among the concepts discussed here are different types of entropy and their bounds, maximal and exponential inequalities, weighted sums, the chaining technique, symmetrization and asymptotic equicontinuity of empirical processes.

Weak convergence and central limit theorems for empirical processes are touched upon only briefly in the short Chapter 6, and applications of this central part of the theory appear only sporadically elsewhere in the book. Instead, the focus is on applications to the almost sure behavior of maximum likelihood and least squares estimators, mainly in nonparametric statistical models. These topics are covered extensively in the six chapters: 4. First Applications: Consistency, 7. Rates of Convergence for Maximum Likelihood Estimators, 9. Rates of Convergence for Least Squares Estimators, 10. Penalties and Sieves, 11. Some Applications to Semiparametric Models, and 12. M-Estimators, with many illustrative examples included. The main aim in this part of the book is to reveal the relations between the asymptotic almost sure behavior of M-type estimators and the complexity of the parameter space of the estimation problem.

It is still noticeable that this text is based on lecture notes prepared for an advanced course in a graduate programm. The author's aim is not to develop a complete abstract theory but to demonstrate the usefulness of basic results from empirical process theory for the study of the asymptotic almost sure behavior of certain selected classes of estimators. This restricted goal is achieved quite well and with reasonable completeness, though the book is not entirely self-contained. The measurability problem (which *is* a problem for empirical processes with large parameter spaces, though of a technical nature) is purposely ignored, for instance, and for most of the models considered, in the examples the existence and computation of maximum likelihood estimators are also not discussed.

On the other hand, there is a guide to the literature in the Notes and Problems and Complements sections at the end of each chapter which helps the beginner to pave his way into the vast amount of published material on empirical processes and their applications. If such a beginner is mainly interested in applications to almost sure results for M-type estimators, then this book will be a good starting-point for him.

Reviewer: E.Häusler (Gießen)

MSC:

62G30 Order statistics; empirical distribution functions
62-02 Research exposition (monographs, survey articles) pertaining to statistics
62G20 Asymptotic properties of nonparametric inference

Keywords:

asymptotic almost sure behaviour of estimators; maximum likelihood estimators; least squares estimators