Panov, E. Yu.

On the theory of generalized entropy solutions of the Cauchy problem for a class of non-strictly hyperbolic systems of conservation laws. (English. Russian original) Zbl 0954.35107

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In the half-plane, $\Pi = \mathbb{R}_+ \times \mathbb{R}$, one considers the Cauchy problem for the system

$$u_t + (\phi(|u|)u)_x = 0, \quad u = u(x,t) \in \mathbb{R}^n$$

with the initial data $u(x,0) = u_0(x) \in L^\infty(\mathbb{R}, \mathbb{R}^n)$ and the hypothesis that $\phi(r) \in C(\mathbb{R}_+)$, and $r\phi(r) \to 0$ for $r \to 0_+$.

For such systems which occur in magnetohydrodynamics in the investigation of the Keyfitz-Kramper isotropic model, the set entropy is defined and described, the concept of generalized entropy solution of the Cauchy problem is introduced and the properties of generalized entropy solutions are studied. The class of strong generalized entropy solutions is distinguished in which the Cauchy problem is uniquely solvable.

If, for $e \in \mathbb{R}^n$ a unit vector, and $0 \leq \delta \leq 1$, $\dot{\Gamma}(e,\delta) := \{u \in \mathrm{Re}^n,\ (u,e) > \delta|u|\} \cup \{0\}$, the condition $u_0(x) \in \dot{\Gamma}(e,0)$ a.e. on $\mathbb{R}$ ensures that the generalized entropy solution is strong and, therefore, unique. Under this condition the convergence of the "vanishing viscosity" method is established.

An example presented in the paper shows that a generalized entropy solution is not necessarily unique in the general case.

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35D05 Existence of generalized solutions of PDE (MSC2000)
35L45 Initial value problems for first-order hyperbolic systems

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