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Derivatives in financial markets with stochastic volatility. (English) Zbl 0954.91025

Cambridge: Cambridge University Press. xiv, 201 p. (2000).

This book presents an approach via asymptotic analysis to the pricing and hedging of derivatives in stochastic volatility models. As explained in the introduction, the central idea is as follows. If volatility, although stochastic, were running extremely fast, the market would behave as in a constant volatility (Black-Scholes) model. Because volatility is running fast but not extremely fast, one can treat the market as a perturbation of a constant volatility Black-Scholes market. The authors show how to do this by an asymptotic analysis and how to compute the resulting correction term.

Thanks to a well-written first chapter on the Black-Scholes theory of derivative pricing, the book is essentially self-contained if one has some basic knowledge in stochastic methods and arbitrage pricing. Its style is largely informal which makes it also accessible to practitioners in the finance industry.

The major part of the book is concerned with the case where the underlying price process X satisfies the generalized geometric Brownian motion stochastic differential equation

$$dX_t = \mu X_t dt + \sigma_t X_t dW_t,$$

where the volatility is of the form $\sigma_t = f(Y_t)$ for some $f > 0$ and Y is an Ornstein-Uhlenbeck process driven by a second Brownian motion possibly correlated with W :

$$dY_t = \alpha(m - Y_t)dt + \beta dW'_t.$$

The crucial property of this model is that Y is ergodic and converges exponentially fast (with rate α) to its invariant distribution κ .

Chapter 4 argues via a statistical analysis of S&P 500 data that a model as above with large α gives a good fit to empirical observations. This provides the motivation for doing an asymptotic analysis for large α or small $\varepsilon = 1/\alpha$.

Chapter 5 presents the resulting asymptotics and gives a formal expansion for European option prices in powers of $\sqrt{\varepsilon}$. The leading term is the Black-Scholes pricing function with volatility $\bar{\sigma}^2 = \int f^2(y)\kappa(dy)$, and the first order term involves the second and third spatial derivatives of this function. This correction term is almost universal in the sense that only the two coefficients in front of the above derivatives depend on the particular model for Y .

The second half of the book first comments in Chapter 6 on the implementation and stability of the basic approach. It then suggests in Chapter 7 hedging strategies whose asymptotically approximate total cost is reduced from order $\sqrt{\varepsilon}$ to order ε by the addition of correction terms, and extends in Chapters 8 and 9 the pricing via asymptotics to some exotic and to American options.

Chapter 10 discusses generalizations; these include an asymptotic analysis of the Merton problem of maximizing expected HARA utility from terminal wealth under stochastic volatility, Markovian volatility models with jumps, martingale techniques and their use to deal with non-Markovian models, and an extension to a multidimensional setting for X and Y .

Finally, Chapter 11 shows how the same methods can be used for interest rate models with stochastic volatility.

Reviewer: [M.Schweizer \(Berlin\)](#)

MSC:

- [91-02](#) Research exposition (monographs, survey articles) pertaining to game theory, economics, and finance
- [91G20](#) Derivative securities (option pricing, hedging, etc.)
- [60H30](#) Applications of stochastic analysis (to PDEs, etc.)
- [35C20](#) Asymptotic expansions of solutions to PDEs

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derivatives; option pricing; stochastic volatility; asymptotic analysis