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Birational Calabi-Yau n -folds have equal Betti numbers. (English) [Zbl 0955.14028](#)

Hulek, Klaus (ed.) et al., New trends in algebraic geometry. Selected papers presented at the Euro conference, Warwick, UK, July 1996. Cambridge: Cambridge University Press. Lond. Math. Soc. Lect. Note Ser. 264, 1-11 (1999).

The author proves that if X and Y are birationally equivalent Calabi-Yau manifolds over \mathbb{C} , then X and Y have the same Betti numbers. This result has now been generalised in a number of different directions [see for example *C.-L. Wang*, *Differ. Geom.* 50, No. 1, 129-146 (1998)], and the ideas of motivic integration of *J. Denef* and *F. Loeser*, *Invent. Math.* 135, No. 1, 201-232 (1999; [Zbl 0928.14004](#))], but this paper gives the very first proof of this result.

The basic idea is as follows. Assume for simplicity that X is defined over \mathbb{Q} ; then one can define a scheme \mathcal{X} over $\text{Spec}(\mathbb{Z})$ such that $\mathcal{X}(\mathbb{C}) = X$. By the Weil conjectures, the Betti numbers of X can be computed by knowing the numbers of rational points of X modulo p^n for all n and a fixed prime p . However, the number of rational points, again by an idea of Weil, can also be computed by integrating certain p -adic measures over $\mathcal{X}(\mathbb{Q}_p)$ where \mathbb{Q}_p denotes the local field of p -adic numbers. The point then is that if X and X' are birational, then they differ only on a set of measure zero with respect to this p -adic measure, and thus the numbers of rational points, and hence the Betti numbers are the same for X and X' , proving the result.

For the entire collection see [[Zbl 0913.00032](#)].

Reviewer: Mark Gross (MR 2000i:14059)

MSC:

- [14J32](#) Calabi-Yau manifolds (algebraic-geometric aspects)
- [14E05](#) Rational and birational maps
- [14F45](#) Topological properties in algebraic geometry
- [14F25](#) Classical real and complex (co)homology in algebraic geometry
- [14E30](#) Minimal model program (Mori theory, extremal rays)
- [14G20](#) Local ground fields in algebraic geometry

Cited in **3** Reviews
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Keywords:

birationally equivalent Calabi-Yau manifolds; Betti numbers; numbers of rational points

Full Text: [arXiv](#)