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Birational Calabi-Yau n-folds have equal Betti numbers. (English) Zbl 0955.14028

Hulek, Klaus (ed.) et al., New trends in algebraic geometry. Selected papers presented at the Euro conference, Warwick, UK, July 1996. Cambridge: Cambridge University Press. Lond. Math. Soc. Lect. Note Ser. 264, 1-11 (1999).

The author proves that if X and Y are birationally equivalent Calabi-Yau manifolds over $\mathbb{C}$, then X and Y have the same Betti numbers. This result has now been generalised in a number of different directions [see for example *C.-L. Wang*, Differ. Geom. 50, No. 1, 129-146 (1998)], and the ideas of motivic integration of *J. Denef* and *F. Loeser*, Invent. Math. 135, No. 1, 201-232 (1999; Zbl 0928.14004)], but this paper gives the very first proof of this result.

The basic idea is as follows. Assume for simplicity that X is defined over $\mathbb{Q}$; then one can define a scheme $\mathcal{X}$ over Spec($\mathbb{Z}$) such that $\mathcal{X}(\mathbb{C}) = X$. By the Weil conjectures, the Betti numbers of X can be computed by knowing the numbers of rational points of X modulo p^n for all n and a fixed prime p. However, the number of rational points, again by an idea of Weil, can also be computed by integrating certain p-adic measures over $\mathcal{X}(\mathbb{Q}_p)$ where $\mathbb{Q}_p$, denotes the local field of p-adic numbers. The point then is that if X and X' are birational, then they differ only on a set of measure zero with respect to this p-adic measure, and thus the numbers of rational points, and hence the Betti numbers are the same for X and X', proving the result.

For the entire collection see [Zbl 0913.00032].

Reviewer: Mark Gross (MR 2000i:14059)

MSC:

14J32	Calabi-Yau manifolds (algebro-geometric aspects)
14E05	Rational and birational maps
14F45	Topological properties in algebraic geometry
14F25	Classical real and complex (co)homology in algebraic geometry
14E30	Minimal model program (Mori theory, extremal rays)
14G20	Local ground fields in algebraic geometry

Keywords:

birationally equivalent Calabi-Yau manifolds; Betti numbers; numbers of rational points

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