The aim of this paper is to extend the theory of quasi-measures in [*J. F. Aarnes*, Adv. Math. 86, No. 1, 41-67 (1991; Zbl 0744.46052)] to locally compact Hausdorff spaces X. For this purpose, the author introduces the notion of a quasi-measure in X and the notion of a quasi-integral on $C_0(X)$ (resp. $C_c(X)$). A real-valued function ρ on $C_0(X)$ (resp. $C_c(X)$) is called a quasi-integral on $C_0(X)$ (resp. $C_c(X)$) if the following conditions are satisfied: (1) $b \geq 0 \Rightarrow \rho(b) \geq 0$ whenever $b \in C_0(X)$ (resp. $C_c(X)$). (2) ρ is linear on $A_0(a)$ (the smallest uniformly closed subalgebra of $C_0(X)$ (resp. $C_c(X)$) containing a) for each $a \in C_0(X)$ (resp. $C_c(X)$). A quasi-integral ρ is said to be bounded if $\sup\{\rho(a) : 0 \leq a \leq 1, a \in C_c(X)\} < +\infty$. Then it is shown that all quasi-integrals on $C_0(X)$ are bounded, a representation of quasi-integrals on $C_c(X)$ in terms of quasi-measures (a generalization of the Riesz representation theorem), and unique extensions of quasi-integrals on $C_c(X)$ to $C_0(X)$.

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