Potapov, M. K.; **Berisha, F. M.**

On the connection between an r-order modulus of smoothness and the best approximation by algebraic polynomials. (Russian. English summary) Zbl 0960.41004

Fundam. Prikl. Mat. 5, No. 2, 563-587 (1999).

Let $1 \leq p < \infty$ and $L_p = L_p[-1,1]$. One denotes by $L_{p,\alpha}$ the space of all functions f such that $f(x)(1-x^2)^\alpha \in L_p$, equipped with the norm $\|f\|_{p,\alpha} = \|f(x)(1-x^2)^\alpha\|_{L_p}$. Let $E_n(f)_{p,\alpha} = \inf_{P_n} \|f - P_n\|_{p,\alpha}$ be the best approximation of f by algebraic polynomials of degree $\leq n-1$. By introducing a non-symmetric generalized translation operator, the authors define a generalized smoothness modulus of order r, denoted by $\widehat{\omega}_r(f,\delta)_{p,\alpha}$. Let p, α and r be given such that $1 \leq p < \infty$, $r \in \mathbb{N}$, $\frac{1}{2} < \alpha \leq 1$ for $p = 1$; $1 - \frac{1}{2} < \alpha < \frac{3}{2} - \frac{1}{2p}$ for $1 < p < \infty$; $1 \leq \alpha < \frac{3}{2}$ for $p = +\infty$, and let $f \in L_{p,\alpha}$. Under these hypotheses one proves that a) for every $\delta \in [0,\pi]$ the inequalities

$$C_1 (\cos^4 \delta/2)^{r(r-1)} K_r(f,\delta)_{p,\alpha} \leq \widehat{\omega}_r(f,\delta)_{p,\alpha} \leq C_2 \frac{1}{(\cos^4 \delta/2)^r} K_r(f,\delta)_{p,\alpha},$$

hold, where $K_r(f,\delta)_{p,d}$ is Peetre's functional and C_1, C_2 are independent of f and δ. b) for every $n \in \mathbb{N}$

$$C_1 E_n(f)_{p,\alpha} \leq \widehat{\omega}_r(f,1/n)_{p,\alpha} \leq C_2 \frac{1}{n^{2r}} \sum_{\nu=1}^{n} \nu^{2r-1} E_\nu(f)_{p,\alpha},$$

where C_1 and C_2 does not depend on f and n.

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MSC:

41A10 Approximation by polynomials
41A17 Inequalities in approximation (Bernstein, Jackson, Nikol'skiĭ-type inequalities)

Keywords:

generalized modulus of smoothness; best approximation

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