Chen, Bang-Yen

Relations between Ricci curvature and shape operator for submanifolds with arbitrary codimensions. (English) Zbl 0962.53015

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As is well known from Nash's immersion theorem that every Riemannian manifold can be immersed in $\mathbb{R}^N$ for N sufficiently great. However, if the immersion has to satisfy additional extrinsic properties, there is no longer any guarantee that such an immersion exists. In the present paper the author investigates the relation between extrinsic invariants (shape operator, length of the mean curvature vector) and intrinsic invariants constructed using sectional curvatures. In that sense it is also a continuation of his previons work [*B.-Y. Chen*, Arch. Math. 60, 568-578 (1993; Zbl 0811.53060)], which in its turn was a source of inspiration for several papers investigating submanifolds of spheres and Lagrangian submanifolds of complex space forms. For this purpose, the author introduces the notion of k-Ricci curvature, which can be seen as the Ricci curvature restricted to a k-dimensional section L_k and introduces a series of intrinsic Riemannian invariants by $\theta_k(p) = \frac{1}{(k-1)} \inf_{L^k,X} \mathrm{Ric}_{L_k}(X)$. Using those invariants several inequalities are obtained between intrinsic and extrinsic invariants of the immersion. In particular it is shown that $H^2(p) \geq \frac{4(n-1)}{n^2}(\frac{\theta_k(p)}{k-1} - c)$, where $2 \leq k \leq n$, for any isometric immersion into a Riemannian space form with constant sectional curvature c. It is also shown that the above inequality, like the other inequalities obtained in the paper, is optimal.

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