

Silverman, Joseph H.

The space of rational maps on \mathbb{P}^1 . (English) [Zbl 0966.14031](#)

Duke Math. J. 94, No. 1, 41-77 (1998).

Introduction: The set of morphisms $\varphi : \mathbb{P}^1 \rightarrow \mathbb{P}^1$ of degree d is parametrized by an affine open subset Rat_d of \mathbb{P}^{2d+1} . In this paper, we consider the action of SL_2 on Rat_d induced by the conjugation action of SL_2 on rational maps; that is, $f \in \text{SL}_2$ acts on φ via $\varphi^f = f^{-1} \circ \varphi \circ f$. The quotient space $M_d = \text{Rat}_d / \text{SL}_2$ arises very naturally in the study of discrete dynamical systems on \mathbb{P}^1 . We prove that M_d exists as an affine integral scheme over \mathbb{Z} , that M_2 is isomorphic to $\mathbb{A}_{\mathbb{Z}}^2$ and that the natural completion of M_2 obtained using geometric invariant theory is isomorphic to $\mathbb{P}_{\mathbb{Z}}^2$. These results, which generalize results of Milnor over \mathbb{C} , should be useful for studying the arithmetic properties of dynamical systems.

MSC:

[14L24](#) Geometric invariant theory

[37K20](#) Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions

[14L30](#) Group actions on varieties or schemes (quotients)

[14E05](#) Rational and birational maps

Cited in **5** Reviews
Cited in **37** Documents

Keywords:

space of rational maps; action of special linear group; discrete dynamical system; geometric invariant theory

Full Text: [DOI](#)

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