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Summary: We study the topological structure of the symmetry group of the standard model, $G_{SM} = U(1) \times SU(2) \times SU(3)$. Locally, $G_{SM} \cong S^3 \times (S^3)^2 \times S^5$. For SU(3), which is an $S^3$-bundle over $S^5$ (and therefore a local product of these spheres) we give a canonical gauge i.e., a canonical set of local trivializations. These formulas give explicitly the matrices of SU(3) without using the Lie algebra (Gell-Mann matrices). Globally, we prove that the characteristic function of SU(3) is the suspension of the Hopf map $S^3@ > h >> S^2$. We also study the case of SU(n) for arbitrary n, in particular the cases of SU(4), a flavor group, and of SU(5), a candidate group of grand unification. We show that the 2-sphere is also related to the fundamental symmetries of nature due to its relation to SO$(3, 1)$, the identity component of the Lorentz group, a subgroup of the symmetry group of several gauge theories of gravity.

MSC:

81R40 Symmetry breaking in quantum theory
81V22 Unified quantum theories
81T13 Yang-Mills and other gauge theories in quantum field theory
22E70 Applications of Lie groups to the sciences; explicit representations
81V15 Weak interaction in quantum theory
81V17 Gravitational interaction in quantum theory
83C45 Quantization of the gravitational field

Keywords:

Gell-Mann matrices; $G_{SM} = U(1) \times SU(2) \times SU(3)$; canonical set of local trivializations; Hopf map $S^3@ > h >> S^2$; grand unification

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