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Orthogonal invariant Riemannian metrics on real Grassmann manifolds. (Russian. English summary) Zbl 0967.53034

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Author's abstract: A full description of the 2-parameter family of all possible $SO(4)$-invariant Riemannian metrics on the real Grassmann manifolds $G_{2,4}$ and $G^+_{2,4}$ is given and an extremal property characterizing the canonical metric on $G^+_{2,4}$ is described. On the basis of these results, we give a new short geometrical proof of the uniqueness (up to the constant factor) of invariant metrics on $G_{p,n}$ and $G^+_{p,n}$ for $(p,n) \neq (2,4)$ and construct these metrics. We use the embeddings of the Grassmann manifolds in the polivector space $\Lambda_{p,n}$ (which can be identified as the Euclidean $\binom{n}{p}$-space), which allows us to solve the problems of intrinsic geometry of Grassmann manifolds by methods of exterior geometry.

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Plücker coordinates; Grassmann algebra; Grassmann manifolds; invariant metrics; exterior geometry