**Alzer, Horst**

**Mean-value inequalities for the polygamma functions.** (English) Zbl 0968.33003

Aequationes Math. 61, No. 1-2, 151-161 (2001).

The polygamma functions are derivatives of the logarithmic derivative $\psi$ of the gamma function. As one of the main results necessary and sufficient conditions are offered for the inequality $|\psi^{(k)}[M^r(\mathbf{x},\mathbf{p})]| \leq M^s(|\psi^{(k)}(\mathbf{x})|,\mathbf{p})$ to hold for all $\mathbf{x}=(x_1,\dots,x_n)\in ]0,\infty[^n$, $\mathbf{p}=(p_1,\dots,p_n)\in\{\mathbf{p}\in ]0,\infty[^n|\ p_1+\dots+p_n=1\}$ $k\geq 1, n\geq 2$ fixed integers), where $M^t(\mathbf{x},\mathbf{p})$ is the $t$-th power mean of $x_1,\dots,x_n$ with weights $p_1,\dots,p_n$ (including the weighted geometric mean if $t=0$). If these conditions connecting $r$ and $s$ are satisfied then there is equality in the above inequality iff $x_1=\dots=x_n$.

Reviewer: János Aczél (Waterloo/Ontario)

**MSC:**

33B15 Gamma, beta and polygamma functions

26D15 Inequalities for sums, series and integrals



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