Shishikura, Mitsuhiro; Lei, Tan

A family of cubic rational maps and matings of cubic polynomials. (English) Zbl 0969.37020

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Let g and f be two monic polynomials of same degree d, and extend each of them to $\mathbb{C} \cup \{\infty \cdot e^{2\pi is}\}$ by mapping $\infty \cdot e^{2\pi is} \mapsto \infty \cdot e^{2\pi ids}$. Glueing these two circles at infinity in opposite directions gives rise to a topological sphere. The *mating* of g and f is the branched covering of this sphere which is given by g on one hemisphere and f on the other. If this map is equivalent to a rational map up to isotopy relative to the post-critical set and topological conjugacy, then g and f are said to be *matable*. In this paper, the authors take g and f cubic polynomials with respectively one double critical point and one period-three orbit containing two simple critical points, a situation which can be reduced to $g(z) = g_a(z) := z^3 + a$ and f one of eight specific polynomials $P_i, \widetilde{P}_i (i = 1, \dots, 4)$, and give criteria for the matability of g_a and P_i in the case where g_a is critically finite. In particular, they exhibit new phenomena which don't occur for quadratic maps, for instance matings which have Thurston obstructions but no Levy cycles. This very nicely written article contains a good survey of Thurston and Levy's theory of branched coverings of the sphere, as well as the general theory of matings. It concludes with an appendix containing experimental observations which give rise to various conjectures.

Reviewer: Line Baribeau (Quebec)

MSC:

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