

**Smeets, Marcus J. E.**

**Parameter shift in GKZ-hypergeometric systems.** (English) [Zbl 0970.33010](#)  
Eindhoven: Eindhoven Univ. of Technology. viii, 113 p. (2000).

The GKZ-hypergeometric system has attracted several interests from algebraic, analytic, and combinatorial viewpoints. It was first introduced in 1989 by Gel'fand, Kapranov, and Zelevinskij. It is a holonomic system formally defined for an integral matrix and a complex vector called the parameters. The systems totally carry a symmetry called the creation operators and the annihilation operators relative to a shift of the parameters. The  $b$ -function, which is a polynomial, describes the precise formula of these operators. It was first introduced by *M. Saito* [Tôhoku Math. J., II. Ser. 44, No. 4, 523-534 (1992; [Zbl 0768.33016](#))]. This article deals with the problem of finding algorithms to construct the  $b$ -functions and the problem of giving explicit formulas for the  $b$ -functions. In the first three chapters, the author gives a neatly arranged survey of the subjects and, in the fourth chapter, he explains applications in discrete optimization such as the Knapsack problem, transportation problem, and matchings. The last chapter treats the Hosten's conjecture related with integer programming. Refer to the paper of *M. Saito, B. Sturmfels* and *N. Takayama* [Compos. Math. 115, No. 2, 185-204 (1999; [Zbl 0936.90038](#))] and a recent book of *M. Saito, B. Sturmfels* and *N. Takayama* [Gröbner deformations of hypergeometric differential equations (2000; [Zbl 0946.13021](#))] for the related materials and another aspect of the GKZ system.

Reviewer: T.Sasaki (Kobe)

**MSC:**

[33C70](#) Other hypergeometric functions and integrals in several variables  
[90C10](#) Integer programming

**Keywords:**

GKZ-hypergeometric system;  $b$ -function