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A multi-parameter interpolation functor and the Lorentz space $L_{p\vec{q}}$, $\vec{q} = (q_1, \dots, q_n)$. (English. Russian original) Zbl 0973.46070

Funct. Anal. Appl. 31, No. 2, 136-138 (1997); translation from Funkts. Anal. Prilozh. 31, No. 2, 79-82 (1997).

Summary: The real interpolation method, which stems from the basic Marcinkiewicz theorem, was introduced by Lons and Peetre. It is described by the functor

$$\Phi_{\theta q}(\varphi) = \left(\int_0^\infty \left(t^{-\theta}\varphi(t)\right)^q \frac{dt}{t} \right)^{1/q}.$$

J. Peetre ["A theory of interpolation of normed spaces", Notes Math. 39 (1968; Zbl 0162.44502)] noticed that under some general conditions on Φ this functor defines an interpolation method that shares many properties of the real method. The central result in this area is the reiteration theorem

$$(\overline{A}_{\Phi_1}, \overline{A}_{\Phi_2})_F = \overline{A}_{F(\Phi_1, \Phi_2)}, \tag{1}$$

which asserts that the interpolation problem for a couple A_{Φ_1}, A_{Φ_2} can be reduced to the interpolation of the parameters Φ_1 and Φ_2 [see *V. I. Demitriev*, and *V. I. Ovchinnikov*, Dokl. Akad. Nauk SSSR 246, 794-797 (1979; Zbl 0432.46067)]. In the present paper, we introduce a functor $\Phi_{\theta\vec{q}}$, $\vec{q} = (q_1, \dots, q_n)$ that generates a many-parameter Lorentz space $L_{p\vec{q}}$. We study interpolation properties of these spaces, which, according to (1), solve the reiteration problem for the corresponding method. The suggested many-parameter interpolation method permits one to describe some finer scales of the Besov spaces $B^\alpha_{p\vec{q}}$ and to refine the bilinear interpolation theorem.

MSC:

46M35 Abstract interpolation of topological vector spaces
46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)

Keywords:

real interpolation method; Marcinkiewicz theorem; reiteration theorem; many-parameter interpolation method; bilinear interpolation theorem

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