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New enumerative results on two-dimensional directed animals. (English) Zbl 0974.05002

An animal $A$ on a graph $G$ is a finite set of vertices of $G$ which are connected in the sense that any two vertices of $A$ are connected by a path in $G$ having all its vertices in $A$. These objects are of interest in statistical mechanics as well as combinatorics, since directed animals are related to directed (site) percolation models. Indeed, as shown by D. Dhar [Equivalence of the two-dimensional directed site animal problem to Baxter’s hard square lattice gas model, Phys. Rev. Lett. 49, 959-962 (1983); Exact solution of a directed-site animals-enumeration problem in three dimensions, Phys. Rev. Lett. 51, No. 10, 853-856 (1983)], enumerating directed animals on a certain graph corresponds to solving a hard particle model on a different graph.

After a thorough review of results, methods and open problems in this field, the author extends Dhar’s idea to show that many of the open questions reduce to finding the position generating function and the (bivariate) perimeter and area generating function for directed animals on square lattices. Moreover, each of these generating functions is essentially the density of a one-dimensional gas model given by the stationary distribution of a probabilistic transition. Although not solved in general, certain special cases give new bivariate generating functions for both square and triangular lattices from which several other generating function results follow as corollaries.

Reviewer: J.R. Galovich (Collegeville)

MSC:
05A15 Exact enumeration problems, generating functions
82B41 Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics
05C99 Graph theory

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References:

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