**Edmunds, David E.**; **Rákosník, Jiří**
**Sobolev embeddings with variable exponent.** (English) Zbl 0974.46040
Stud. Math. 143, No. 3, 267-293 (2000).

Let $\Omega$ be a bounded open subset of $\mathbb{R}^n$ with Lipschitz boundary and let $p : \overline{\Omega} \to [1, \infty)$ be Lipschitz-continuous. The authors consider the generalised Lebesgue space $L^{p(x)}(\Omega)$ and the corresponding Sobolev space $W^{1,p(x)}(\Omega)$, consisting of all $f \in L^{p(x)}(\Omega)$ with first-order distributional derivatives in $L^{p(x)}(\Omega)$.

It is shown that if $1 \leq p(x) < n$ for all $x \in \Omega$, then there is a constant $c > 0$ such that for all $f \in W^{1,p(x)}(\Omega)$,

$$\|f\|_{M,\Omega} \leq c\|f\|_{1,p,\Omega}.$$

Here $\|\cdot\|_{M,\Omega}$ is the norm on an appropriate space of Orlicz-Musielak type and $\|\cdot\|_{1,p,\Omega}$ is the norm on $W^{1,p(x)}(\Omega)$. The inequality reduces to the usual Sobolev inequality if $\sup_\Omega p < n$. Corresponding results are proved for the case in which $p(x) > n$ for all $x \in \Omega$.

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**MSC:**

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
26D10 Inequalities involving derivatives and differential and integral operators



**Keywords:**

generalised Lebesgue spaces; embeddings; extension operators; Sobolev space; space of Orlicz-Musielak type; Sobolev inequality

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