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Transformation of measure on Wiener space. (English) Zbl 0974.46044
Springer Monographs in Mathematics. Berlin: Springer. xiii, 297 p. (2000).

The book is devoted to transformations of Wiener measures on the abstract Wiener space relative to different types of mappings including shifts, rotations and flows. For this, mappings of Sobolev type $\mathbb{D}_{p,k}$ in Malliavin's sense of quasi-sure analysis, satisfying definite conditions, are used. There are given many results such as the classical ones of Cameron and Martin and also recent ones including investigations of the authors. The crucial role plays the Girsanov theorem and Theorem 2.7.1 that the class of measures representable by shifts ($T = I_w + u$, where u is an H -valued random variable for an abstract Wiener space (W, H, μ)) is dense under the total variation norm in the class of probability measures on W which are equivalent to μ . Applications to the Sard inequality and infinite-dimensional flows are given.

For example, in accordance with Theorem 5.3.1 for definite flows φ_{st} of the form

$$\varphi_{st}(\omega) = \omega + \int_s^t X_r(\varphi_{sr}(\omega)) dr,$$

where X is a measurable map from $\mathbb{R}_+ \times W$ in H , $0 \leq s \leq t \leq T$, $r \in \mathbb{R}_+$, the Radon-Nikodým density is given by the following equation

$$\frac{d\varphi_{st}^*\mu}{d\mu} = \exp\left(\int_s^t (\delta X_r)(\psi_{rt}) dr\right),$$

where ψ_{st} is an inverse flow, δX_r is the first-order Wiener integral.

Generalized Radon-Nikodým derivatives of measures are studied with the help of the Rademacher class of Wiener functionals and the Ornstein-Uhlenbeck semigroup.

One of the chapters contains results of the authors about a class of mappings on W preserving the Wiener measure. Finally, a measure theoretic degree on the Wiener space is presented. Its relations with the Leray-Schauder degree and applications to absolute continuity are outlined.

Reviewer: [Sergey Lüdkovsky \(Moskva\)](#)

MSC:

- 46G12 Measures and integration on abstract linear spaces
- 46-02 Research exposition (monographs, survey articles) pertaining to functional analysis
- 28-02 Research exposition (monographs, survey articles) pertaining to measure and integration
- 60-02 Research exposition (monographs, survey articles) pertaining to probability theory
- 60H05 Stochastic integrals
- 60G15 Gaussian processes
- 60G30 Continuity and singularity of induced measures
- 28C20 Set functions and measures and integrals in infinite-dimensional spaces (Wiener measure, Gaussian measure, etc.)
- 60H25 Random operators and equations (aspects of stochastic analysis)
- 60G35 Signal detection and filtering (aspects of stochastic processes)
- 46T12 Measure (Gaussian, cylindrical, etc.) and integrals (Feynman, path, Fresnel, etc.) on manifolds
- 60H07 Stochastic calculus of variations and the Malliavin calculus

Cited in **2** Reviews
Cited in **12** Documents

Keywords:

transformation of Wiener measure; abstract Wiener space; stochastic control theory; Malliavin calculus; canonical Gaussian cylinder set measure; Radon-Nikodým derivative; Ramer's formula; Girsanov theorem;

infinite-dimensional flows; Rademacher class of Wiener functionals; shifts; rotations; quasi-sure analysis; Sard inequality; Radon-Nikodým derivatives; Ornstein-Uhlenbeck semigroup