Christodoulou, S.

Initially κ-compact spaces for large κ. (English) Zbl 0976.54022

Commentat. Math. Univ. Carol. 40, No. 2, 319-325 (1999).

Let $X = \langle X, \tau_X \rangle$ be a T_2 space. The closed pseudocharacter $\psi_c(X)$ of X is defined as $\sup\{\psi_c(p, X)\, p \in X\} + w$, where $\psi_c(p, X) = \min\{|V|\, V \subseteq \tau_X, p \in \bigcap V, \bigcap\{\overline{V}\, p \in V\} = \{p\}\}$.

The author gives a new bound for $\psi_c(X)$ $\psi_c(X) \leq 2^{d(X)}$ and for a Uryson space X $\psi_c \leq 2^{s(X)}$, where $d(X)$ and $s(X)$ are common cardinals for topological spaces [see, e.g., *I. Juhász*, Cardinal functions in topology – ten years later, Mathematical Centre Tracts 123 (1980; Zbl 0479.54001)].

If every open cover of the space X of size k ($k \geq w$ is a cardinal) has a finite subcover, then X is said to be initially k-compact. If for every $A \subseteq X$ with $|A| \leq k$ there is $Y \subseteq X$, Y compact such that $A \subseteq Y$, then X is called k-bounded. Using the first bound for $\psi_c(X)$ it is proved in this paper that each T_2 space X initially k-compact is λ-bounded for every cardinal λ with $2^\lambda \leq k$.

This result improves the known statement for T_3 spaces and it is used for the proposition on initially k-compact T_2 spaces being compact for special cardinal k.

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MSC:

54D20 Noncompact covering properties (paracompact, Lindelöf, etc.)
54A25 Cardinality properties (cardinal functions and inequalities, discrete subsets)
54D10 Lower separation axioms (T_0–T_3, etc.)

Keywords:

initially κ-compact space; κ-bounded space; closed pseudocharacter for a T_2 space

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