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Relaxation of some multi-well problems. (English) Zbl 0977.74029

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Summary: Mathematical models of phase transitions in solids lead to the variational problem, minimize $\int_\Omega W(Du)dx$, where W has a multi-well structures, i.e. $W = 0$ on a multi-well set K and $W > 0$ otherwise. We study this problem in two dimensions in the case of equal determinants, i.e. for $K = \mathrm{SO}(2)U_1 \cup \dots \cup \mathrm{SO}(2)U_k$ or $K = \mathrm{O}(2)U_1 \cup \dots \cup \mathrm{O}(2)U_k$ for $U_1, \dots, U_k \in \mathbb{M}^{2\times 2}$ with $\det U_i = \delta$, in three dimensions when the matrices U_i are essentially two-dimensional, and also in the case $K = \mathrm{SO}(3)\widehat{U}_1 \cup \dots \cup \mathrm{SO}(3)\widehat{U}_k$ for $U_1, \dots, U_k \in \mathbb{M}^{3\times 3}$ with $(\operatorname{adj} U_i^T U_i)_{33} = \delta^2$, which arises in the study of thin films. Here $\widehat{U}_i$ denotes the (3×2) matrix formed with the first two columns of U_i. We characterize generalized convex hulls, including the quasiconvex hull, of these sets, prove existence of minimizers, and identify conditions for the uniqueness of the minimizing Young measure. Finally, we use the characterization of the quasiconvex hull to propose 'approximate relaxed energies', quasiconvex functions which vanish on the quasiconvex hull of K and grow quadratically away from it.

MSC:

74G65 Energy minimization in equilibrium problems in solid mechanics
74N99 Phase transformations in solids

Keywords:

approximate relaxed energies; phase transitions in solids; variational problem; multi-well structure; generalized convex hulls; quasiconvex hull; existence of minimizers; uniqueness; Young measure

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