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Primary modules over commutative rings. (English) Zbl 0979.13003

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Let R be a commutative ring. All modules considered are unital R-modules. For an ideal I of R and for a submodule N of an R-module M the following sets are defined:

$$\sqrt{I} = \{r \in R : r^n \in I \text{ for some positive integer } n\},$$

$$(N : M) = \{r \in R : rM \subseteq N\},$$

$$E_M(N) = \{rm : r \in R,\ m \in M \text{ and } r^k m \in N \text{ for some positive integer } k.$$

By $RE_M(N)$ will be denoted the submodule of M generated by the non-empty subset $E_M(N)$ of M. – A submodule N of M is called prime (respectively, primary) if $N \neq M$ and whenever $r \in R$, $m \in M$ and $rm \in N$ then $m \in N$ or $r \in (N : M)$ (respectively, $r \in \sqrt{(N : M)}$). The module M will be called primary if its zero submodule is primary. For any submodule N of an R-module M, the radical, $\mathrm{rad}_M(N)$, of N is defined to be the intersection of all prime submodule of M containing N and $\mathrm{rad}_M(N) = M$ if N is not contained in any prime submodules of M. The radical of the module M is defined to be $\mathrm{rad}_M(0)$.

The author gives the definition that the module M satisfies the radical formula for primary submodules if $\mathrm{rad}_M(N) = RE_M(N)$ for every primary submodule N of M.

The main result is: If R is a commutative domain which is either Noetherian or a UFD then R is one-dimensional if and only if every (finitely generated) primary R-module has prime radical, and this holds precisely when every (finitely generated) R-module satisfies the radical formula for primary submodules.

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13A10	Radical theory on commutative rings (MSC2000)
13C05	Structure, classification theorems for modules and ideals in commutative rings
13A15	Ideals and multiplicative ideal theory in commutative rings

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