Bezhaev, A. Yu.; **Vasilenko, V. A.**

Variational theory of splines. (English) Zbl 0979.41007

Dordrecht: Kluwer Academic/Plenum Publishers. xvii, 280 p. (2001).

It is well known that the variational spline theory, the object of this monograph, is today a well-developed field in approximation theory. Plenty of the new significant results are presented in this book in brief and complete form. The authors aim their book for specialists in numerical analysis and also for students in pure and applied mathematics. The referee also hopes that this volume will serve as a powerful impulse in theoretical studies of splines, new applications, and software developments. The authors summarize the content of the book in their introduction very clearly and itemized, we abbreviate it in the following lines. In Chapter 1 the definitions are given in Atteia style for the case of abstract Hilbert space, and some theorems, among others. Chapter 2 is devoted to the characterization of splines. Chapter 3 contains general convergence techniques for interpolating slines in the Hilbert space and error estimation for splines in Sobolev spaces. In Chapter 4 the finite dimensional analogues of interpolating and smoothing splines are considered, the general convergence theorems are proved and a special technique for error estimation is presented. Questions concerning the spline-approximation of discontinuous functions are also considered. In Chapter 5 first the fundamental inequalities for the functions from Sobolev spaces with condense zeros are treated, later the detailed consideration of multidimensional B-splines as finite elements for the construction of the analogues of D^m-splines at the scattered meshes is found. In Chapter 6 a new object in variational spline theory is considered: the traces of D^m-splines on the manifolds. This aims to find the numerical algorithm for the interpolation of the function which is known in the interpolation points of scattered condense mesh on the manifold. The second part of the chapter is connected with the approximation in analytical and finite element forms of function given on manifold by the trace of D^m-spline from the "thin layer" near the manifold. Chapter 7 is devoted to vector splines. In Chapter 8 tensor and blending splines are presented. Chapter 9 is devoted to optimal approximation of operators and functionals. Chapter 10 summarizes the descriptions of variational spline objects and methods presented in previons chapters. This chapter gives the principle of the classification on groups. Chapter 11 is devoted to so-called sum-product approximations. General consideration for the abstract problem of optimal $\sum\prod$-approximation in the tensor product of two Hilbert spaces is given in the numerical finite dimensional sense. Numerical results are also given. Chapter 12 presents, among others, various tricks to accelerate the calculations in Newton's method. The book contains two appendices. The first one holds the main theorems on functional analysis, the second one contains the brief description of software LIDA-3 which was produced by the authors and their colleagues in Novosibirsk. The book is recommended to everyone who is interested in the subject.

Reviewer: Laszlo Leindler (Szeged)

MSC:

41A15 Spline approximation
65D99 Numerical approximation and computational geometry (primarily algorithms)