Gebbie, Tim; Ellis, G. F. R.

1 + 3 covariant cosmic microwave background anisotropies. I: Algebraic relations for mode and multipole expansions. (English) Zbl 0979.83048


This is the first in a series of articles aiming at a systematic treatment of cosmic microwave background anisotropies in a 1 + 3 covariant and gauge-invariant fashion. It is a characteristic feature of this approach that it is based on 1 + 3 covariantly defined geometric quantities on a general-relativistic spacetime, rather than on quantities which are introduced as small perturbations around some background from the outset.

The 35 pages of this first paper are devoted to algebraic fundamentals, i.e., the dynamical equations are not yet considered. The cosmic microwave background is mathematically modeled in terms of a temperature function $T(x^i, e^a)$, defined on a spacetime manifold with a prescribed 4-velocity field $u^a$, where $T$ depends on the spacetime point $x^i$ and on the spatial direction indicated by a unit vector $e^a$ perpendicular to $u^a$. This setting is motivated by the observational fact that the photons of the cosmic microwave background radiation come to us with a Planckian spectrum, where the temperature parameter depends on the spatial direction, and that it is reasonable to assume that the same is true for observers at other positions in the universe. As the mathematical tools for analyzing such function of $x^i$ and $e^a$ the authors discuss in full detail multipole expansions with respect to $e^a$ and mode expansions (generalized spatial Fourier expansions) with respect to $x^i$.

A great part of the paper is devoted to statistical aspects, presupposing not a single temperature function but rather an ensemble of temperature functions. This is a way of treating statistical perturbations of temperature functions. The analysis is centered upon angular two-point correlation functions which leads to the so-called angular power spectrum. The case of homogeneous and isotropic Gaussian perturbations is treated in particular.

The paper can be viewed as a self-contained introduction into the subject. Some of the mathematical techniques, e.g. the multipole expansion techniques, are of interest even beyond the special physical application the paper is aimed at.

For Part II, see the following review Zbl 0979.83044.

Reviewer: Volker Perlick (Berlin)

MSC:

83F05 Relativistic cosmology
85A40 Astrophysical cosmology
83B05 Observational and experimental questions in relativity and gravitational theory

Keywords:

cosmic microwave background anisotropies; gauge-invariant; temperature function; spatial Fourier expansions

Full Text: DOI arXiv

References:
