Huang, Jing-Song; **Pandžić, Pavle**

Dirac cohomology, unitary representations and a proof of a conjecture of Vogan. (English)

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In the present paper a proof is given of Vogan's conjecture on Dirac cohomology.

Let G be a connected semisimple Lie group with finite centre, K the maximal subgroup of G corresponding to the Cartan involution θ, with Cartan decomposition $g = k \oplus p$ and let g^C be the complexification of g. If X is an irreducible unitarizable (g^C, K)-module, the Dirac operator D acts on $X \otimes S$, where S is the space of spinors for p. The Vogan conjecture states that if D has nonzero kernel on $X \otimes S$, then the infinitesimal character of X can be described in terms of the heighest weight of a $\widetilde{K}$-type in $\ker D$, where $\widetilde{K}$ is a double cover of K corresponding to the group $Spin(p)$.

The main new idea for proving Vogan's conjecture is introducing a differential d on the K-invariants in $U(g^C) \otimes C(p^C)$ related to D, where $U(g^C)$ is the universal enveloping algebra of g^C and $C(p^C)$ is the Clifford algebra of the complexification of p. The conjecture follows by determining the cohomology of d.

Reviewer: Sergio Console (Torino)

MSC:

22E46 Semisimple Lie groups and their representations
22E47 Representations of Lie and real algebraic groups: algebraic methods (Verma modules, etc.)

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