Jarnicki, Witold
Kobayashi-Royden vs. Hahn pseudometric in $\mathbb{C}^2$. (English) Zbl 0982.32012
Ann. Pol. Math. 75, No. 3, 289-294 (2000).

For a domain in $D \subset \mathbb{C}^n$, the Kobayashi-Royden pseudometric k_D and the Hahn pseudometric h_D are defined, by the formulas

$$k_D(z;X) = \inf\{|\alpha| : f(0) = z, \alpha f'(0) = X\},$$

$$h_D(z;X) = \inf\{|\alpha| : f(0) = z, \alpha f'(0) = X, f \text{ is injective}\},$$

for $z \in D, X \in \mathbb{C}^n$, here the inf is taken for holomorphic maps $f : \triangle \to D$, where $\triangle$ is the unit disc. Obviously $k_D \leq h_D$.

It is known that for a domain $D \subset \mathbb{C}$, $k_D = h_D$ if and only if D is simply connected. Overholt showed that for any domain $D \subset \mathbb{C}^n$, with $n \geq 3$, $k_D = h_D$ [*M. Overholt*, Ann. Pol. Math. 62, No. 1, 79-82 (1995; Zbl 0847.32027)]. This paper proves the following result: Let $D_1, D_2 \subset \mathbf{C}$ be domains. Then: (1) If at least one of D_1, D_2 is simply connected, then $k_{D_1\times D_2} \equiv h_{D_1\times D_2}$. (2) If at least one of D_1, D_2 is biholomorphic to $\mathbb{C}_*$, then $k_{D_1\times D_2} \equiv h_{D_1\times D_2}$. (3) Otherwise $k_{D_1\times D_2} \not\equiv h_{D_1\times D_2}$.

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MSC:

32F45 Invariant metrics and pseudodistances in several complex variables

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Hahn pseudometric; Kobayashi pseudometric

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