Chipot, M.; **Lovat, B.**

Existence and uniqueness results for a class of nonlocal elliptic and parabolic problems. (English) Zbl 0984.35066

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The authors study some class of nonlinear nonlocal elliptic and parabolic problems. Precisely, let Ω be a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂Q, and let Γ_0 be a subset of $\partial\Omega$ having a positive superficial measure. Set $V = \{v \in H^1(\Omega) \mid v = 0 \text{ on } \Gamma_0\}$, and consider m functionals $q_1, \dots, q_m : V \to \mathbb{R}$, where each q_i is a positive homogeneous function of degree $\alpha_i (\in \mathbb{R})$. For a positive function $a : \mathbb{R}^m \to \mathbb{R}$, consider the problem:

$$-a\big(q_1(u), \dots, q_m(u)\big) Au = f \text{ in } \Omega, \tag{1}$$

$$u = 0 \text{ on } \Gamma_0,\ \partial_\nu u = 0 \text{ on } \partial\Omega \setminus \Gamma_0, \tag{2}$$

where f is an element of V', the dual of V, A is a linear elliptic operator in divergence form, and $\partial_\nu u$ denotes the conormal derivative of u. It is assumed that the bilinear form canonically associated to A is coercive on V. A typical example of A is the Laplacian Δ. It is shown that this problem has as many solutions as the system of equations in $\mu = (\mu_1, \dots, \mu_m) \in \mathbb{R}^m$:

$$a^{\alpha_i}(\mu)\mu_i = q_i(\varphi),\ i = 1, \dots, m,$$

where $\varphi \in V$ is the unique solution of the problem:

$$-A\varphi = f \text{ in } \Omega, \quad \varphi = 0 \text{ on } \Gamma_0,\ \partial_\nu \varphi = 0 \text{ on } \partial\Omega \setminus \Gamma_0.$$

When $\Gamma_0 = \partial\Omega$ and $V = H_0^1(\Omega)$, also the parabolic problem associated to problem (1)-(2) is solved and in particular it is shown that the solutions of the parabolic problem can quench in the sense that they can vanish identically at some finite time.

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MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35K60 Nonlinear initial, boundary and initial-boundary value problems for linear parabolic equations

Keywords:

nonlocal problem; elliptic boundary value problem; quenching