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Excitation thresholds for nonlinear localized modes on lattices. (English) Zbl 0984.35147

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Summary: We consider spatially localized and time-periodic solutions to discrete extended Hamiltonian dynamical systems (coupled systems of infinitely many 'oscillators' which conserve total energy). These play a central role as carriers of energy in models of a variety of physical phenomena. Such phenomena include nonlinear waves in crystals, biological molecules and arrays of coupled optical waveguides. In this paper we study excitation thresholds for (nonlinearly dynamically stable) ground state localized modes, sometimes referred to as 'breathers', for networks of coupled nonlinear oscillators and wave equations of nonlinear Schrödinger (NLS) type. Excitation thresholds are rigorously characterized by variational methods. The excitation threshold is related to the optimal (best) constant in a class of discrete interpolation inequalities related to the Hamiltonian energy. We establish a precise connection among d, the dimensionality of the lattice, $2\sigma+1$, the degree of the nonlinearity, and the existence of an excitation threshold for discrete nonlinear Schrödinger systems (DNLS). We prove that if $\sigma\geq 2/d$, then ground-state standing waves exist if and only if the total power is larger than some strictly positive threshold, $\nu_{\text{thresh}}(\sigma,d)$. This proves a conjecture of *S. Flach, K. Kladko* and *R. S. MacKay* [Phys. Rev. Lett. 78, No. 7, 1207-1210 (1997)] in the context of DNLS. We also discuss upper and lower bounds for excitation thresholds for ground states of coupled systems of NLS equations, which arise in the modelling of pulse propagation in coupled arrays of optical fibres.

MSC:

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78A60 Lasers, masers, optical bistability, nonlinear optics

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discrete extended Hamiltonian dynamical systems; discrete nonlinear Schrödinger systems; upper and loweer bounds; modelling of pulse propagation

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