De Jeu, Rob

Towards regulator formulae for the K-theory of curves over number fields. (English)

Zbl 0985.19002
Compos. Math. 124, No. 2, 137-194 (2000).

The author computes the Beilinson regulator on a subgroup of the motivic cohomology group $H^2_{\mathcal{M}}(F,\mathbb{Q}(n+1)) = K^{(n+1)}_{2n}(F)$, where F is the function field of a complete smooth geometrically irreducible curve $\mathcal{C}$ over a number field, under the Beilinson-Soulé conjecture on weights, by using the motivic complexes $\widetilde{\mathcal{M}}^{\bullet}_{(m)}$ that he constructed in a previous article [*R. De Jeu*, Compos. Math. 96, No. 2, 197-247 (1995; Zbl 0868.19002)].

More precisely, let $\mathcal{C}$ be a smooth geometrically irreducible projective curve over a number field k, with function field $F = k(\mathcal{C})$, and let $\mathbb{Q}\otimes_{\mathbb{Z}} K_m(\mathcal{C}) = \bigoplus_{n=1}^{m+1} K^{(n)}_m(\mathcal{C})$ be the decomposition of the K-theory with rational coefficients, given by the eigenspaces under Adams operators. There are regulator maps to Deligne cohomology $K^n_m(\mathcal{C}) \to H^{2n-m}_{\mathcal{D}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(n))$, where $\mathcal{C}_{\mathrm{an}}$ is the analytic manifold associated to $\mathbb{C}\otimes_{\mathbb{Q}}\mathcal{C}$ and $\mathbb{R}(n) = (2\pi i)^n\mathbb{R} \subset \mathbb{C}$. For $n \geq 2$ and $m = 2n-2$, this gives a map with values in $H^2_{\mathcal{D}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(n)) \simeq H^1_{\mathrm{dR}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(n-1))$; and in fact in the subspace $H^1_{\mathrm{dR}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(n-1))^+$ of the forms ψ satisfying the identity $\psi\circ\sigma = \overline{\psi}$, where σ denotes the canonical involution on $\mathcal{C}_{\mathrm{an}}$. So, by wedging with an holomorphic 1-form on $\mathcal{C}_{\mathrm{an}}$ and integrating, one lands in $\mathbb{R}(1)$.

In the paper under review the author uses the quotient complexes $\widetilde{\mathcal{M}}^{\bullet}_{(m)}$ that he introduced in a previous work [op. cit.] and the Beilinson-Soulé conjecture for fields of characteristic 0 in order to construct symbols $[f]_n\otimes g$ in $K^{(n+1)}_{2n}(F)$; then he proves that their images under the map $H^2(\widetilde{\mathcal{M}}^{\bullet}_{(n+1)}(F)) \to K^{(n+1)}_{2n}(F) \xrightarrow{\text{reg}} H^1_{\mathrm{dR}}(F;\mathbb{R}(n))^+ \to \mathbb{R}(1)$ has the form conjectured by *A. B. Goncharov* ["Polylogarithms in arithmetic and geometry", Proc. Internat. Congr. Math. (Zürich), 374-387 (1994; Zbl 0849.11087)]. In addition, for $n = 2$ and $n = 3$, he observes that the map above exists without assumption.

The author also studies compatibility of Gersten complexes for $\widetilde{\mathcal{M}}_{(m)}$ and K-theory on $\mathcal{C}$. Most proofs in the paper are general, but unfortunately the combinatorics become more and more complicated as n increases. So the author restricts his final computations to $K^{(3)}_4$ and $K^{(4)}_6$. Finally he shows that $K^{(3)}_4(\mathcal{C})$ (resp. $K^{(4)}_6(\mathcal{C})$), $H^2(\widetilde{\mathcal{M}}^{\bullet}_{(3)}(\mathcal{C}))$ (resp. $H^2(\widetilde{\mathcal{M}}^{\bullet}_{(4)}(\mathcal{C}))$) and Goncharov's version $H^2(\Gamma'(\mathcal{C},3))$ (resp. $H^2(\Gamma'(\mathcal{C},4))$) of the latter group all have the same image in $H^1_{\mathrm{dR}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(2))^+$ (resp. in $H^1_{\mathrm{dR}}(\mathcal{C}_{\mathrm{an}};\mathbb{R}(3))^+$) under the regulator map.

Reviewer: Jean-François Jaulent (Talence)

MSC:

19F27 Étale cohomology, higher regulators, zeta and L-functions (K-theoretic aspects)
11G30 Curves of arbitrary genus or genus $\neq 1$ over global fields
19D45 Higher symbols, Milnor K-theory
19E08 K-theory of schemes

Keywords:

K-theory; localization; boundary map; curves; number field; regulator; motivic cohomology; Beilinson-Soulé conjecture; symbols

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