Paluszyński, Maciej; Šikić, Hrvoje; Weiss, Guido; Xiao, Shaoliang
Generalized low pass filters and MRA frame wavelets. (English) Zbl 0985.42020
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Let $\psi \in L^2(R)$ and define the wavelet system $\{\psi_{j,k}\}_{j,k\in Z}$ by $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$. The function ψ is called a tight (normalized) frame wavelet (TFW) if $\sum_{j,k\in Z} |\langle f, \psi_{j,k}\rangle|^2 = \|f\|^2$ for all $f \in L^2(R)$; this is equivalent to

$$f = \sum_{j,k\in Z} \langle f, \psi_{j,k}\rangle \psi_{j,k}, \forall f \in L^2(R).$$

Tight frame wavelets generalize wavelets in the sense that $\{\psi_{j,k}\}_{j,k\in Z}$ might not be linearly independent, i.e., the wavelet system can be redundant. A 2π-periodic function m on R is called a generalized filter if $|m(\xi)|^2 + |m(\xi + \pi)|^2 = 1$ a.e.; and a function $\varphi \in L^2(R)$ is a pseudo-scaling function if there exists a generalized filter m such that $\widehat{\varphi}(2\xi) = m(\xi)\widehat{\psi}(\xi)$. Finally, a TFW ψ is called a MRA TFW if there exists a pseudo-scaling function φ with associated filter m such that

$$\widehat{\psi}(\xi) = e^{i\xi/2}\overline{m(\xi/2 + \pi)}\widehat{\varphi}(\xi/2).$$

Using a subclass of the generalized filters, a procedure for construction of all MRA TFWs is given. The results are compared to frame multiresolution analysis as defined by Benedetto et al., and it is shown that the MRA TFWs contain the tight frames by the latter authors (however, frame multiresolution analysis also yields a construction of non-tight frames). An extension of the orthogonalization trick by Daubechies is proved: if $\{\varphi(\cdot - k)\}_{k\in Z}$ is a frame for its closed linear span V_0, then there exists a function $\widetilde{\varphi} \in V_0$ such that $\{\widetilde{\varphi}(\cdot - k)\}_{k\in Z}$ is a tight frame for V_0. The class of TFW multipliers is characterized and shown to be equal to the class of MRA TFW multipliers.

Reviewer: Ole Christensen (Lyngby)

MSC:

42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems

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frame; tight frame wavelet; MRA tight wavelet frame; multiresolution analysis

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