**Gazzola, Filippo; Serrin, James; Tang, Moxun**

**Existence of ground states and free boundary problems for quasilinear elliptic operators.**

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Summary: The authors prove the existence of nonnegative nontrivial solutions of the quasilinear equation $\Delta_m u + f(u) = 0$ in $\mathbb{R}^n$ and of its associated free boundary problem, where $\Delta_m$ denotes the $m$-Laplace operator. The nonlinearity $f(u)$, defined for $u > 0$, is required to be Lipschitz-continuous on $(0, \infty)$, and in $L^1$ on $(0, 1)$ with $\int_0^u f(s)ds < 0$ for small $u > 0$; the usual condition $f(0) = 0$ is thus completely removed. When $n > m$, existence is established essentially for all subcritical behavior of $f$ as $u \to \infty$, and, with some further restrictions, even for critical and supercritical behavior. When $n = m$ we treat various exponential growth conditions for $f$ as $u \to \infty$, while when $n < m$ no growth conditions of any kind are required for $f$. The proof of the main results moreover yield as a byproduct an a priori estimate for the supremum of a ground state in terms of $n, m$ and elementary parameters of the nonlinearity. The results are thus new and unexpected even for the semilinear equation $\Delta u + f(u) = 0$.

The proofs use only straightforward and simple techniques from the theory of ordinary differential equations; unlike well-known earlier demonstrations of the existence of ground states for the semilinear case, the authors rely neither on critical point theory nor on the Emden-Fowler inversion technique.

**MSC:**

35J70 Degenerate elliptic equations
35J60 Nonlinear elliptic equations



**Keywords:**

degenerate $m$-Laplace operator; exponential growth conditions; a priori estimate