

**Gehrke, Mai; Harding, John**

**Bounded lattice expansions.** (English) Zbl 0988.06003  
J. Algebra 238, No. 1, 345-371 (2001).

The notion of a canonical extension of a lattice (with additional operations, respectively) is introduced. A canonical extension of a lattice  $L$  is a pair  $(e, C)$ , where  $C$  is a complete lattice,  $e : L \rightarrow C$  is a lattice embedding such that:

- i) every element  $c \in C$  can be expressed both as a join of meets and a meet of joins of elements from  $e(L)$  (the image of  $L$ ),
- ii) whenever  $A \subseteq C$  is a set of elements which are joins of elements from  $e(L)$  and  $B$  is a set of elements which are meets of elements from  $e(L)$ , then  $\bigwedge A \leq \bigvee B$  iff  $\bigwedge A^* \leq \bigvee B^*$  for some finite subsets  $A^* \subseteq A$  and  $B^* \subseteq B$ .

Every lattice has a uniquely determined canonical extension. Both a concrete description and an abstract characterization of this extension is given. Various results involving the preservation of identities under canonical extension are obtained.

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**MSC:**

**06B05** Structure theory of lattices  
**06B23** Complete lattices, completions

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canonical extension of a lattice; complete lattice; lattice embedding; preservation of identities

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