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**Formal GNS construction and states in deformation quantization.** (English) Zbl 0989.53057

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Summary: In this paper we develop a method of constructing Hilbert spaces and the representation of the formal algebra of quantum observables in deformation quantization which is an analog of the well-known GNS construction for complex $C^*$-algebras: in this approach the corresponding positive linear functionals ("states") take their values not in the field of complex numbers, but in (a suitable extension field of) the field of formal complex Laurent series in the formal parameter. By using the algebraic and topological properties of these fields we prove that this construction makes sense and show in physical examples that standard representations such as the Bargmann and Schrödinger representation come out correctly, both formally and in a suitable convergence scheme. For certain Hamiltonian functions (contained in the Gel'fand ideal of the positive functional) a formal solution to the time-dependent Schrödinger equation is shown to exist. Moreover, we show that for every Kähler manifold equipped with the Fedosov star product of Wick type all the classical delta functionals are positive and give rise to some formal Bargmann representation of the deformed algebra.

**MSC:**

53D55 Deformation quantization, star products
81S10 Geometry and quantization, symplectic methods
46L60 Applications of selfadjoint operator algebras to physics



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