Aktosun, Tuncay; **Klaus, Martin**

Small-energy asymptotics for the Schrödinger equation on the line. (English) Zbl 0989.81129

Inverse Probl. 17, No. 4, 619-632 (2001).

The authors consider the one-dimensional Schrödinger equation

$$-\psi''(k,x)+V(x)\psi(k,x)=k^2\psi(k,x)$$

on the line, where V is assumed to have a first moment. Solutions f of this equation, which satisfy

$$e^{ikx}f_l(k,x)=1+o(1),\quad e^{-ikx}f_l'(k,x)=ik+o(1)\quad \text{for } x\to\infty$$

and

$$e^{ikx}fr(k,x)=1+o(1),\quad e^{ikx}fr'(k,x)=-ik+o(1)\quad \text{for } x\to-\infty$$

are called the left and right Jost solutions. They are important because scattering, reflection and transmission of waves can be expressed in terms of these functions. Their main result, for which two independent proofs are given, states for the small energy asymptotics:

Theorem. Assume V is real valued and has a first moment. For any $x\in\mathbb{R}$ the Jost solutions f_α ; $\alpha=l,r$ satisfy

1. If $f_\alpha(0,x)\neq 0$ then $\frac{f_\alpha'(k,x)}{f_\alpha(k,x)}=\frac{f_\alpha'(0,x)}{f_\alpha(0,x)}+\varepsilon(\alpha)\frac{ik}{f_l(0,x)^2}+o(k)$.

2. If $f_\alpha'(0,x)\neq 0$ then $\frac{f_\alpha(k,x)}{f_\alpha'(k,x)}=\frac{f_\alpha(0,x)}{f_\alpha'(0,x)}-i\varepsilon(\alpha)\frac{k}{f_\alpha'(0,x)^2}+o(k)$ with $k\to 0$ in the upper half plane and $\varepsilon(l)=+1$, $\varepsilon(r)=-1$. If V has a second moment these results can be improved. The proofs are mainly based on estimates of the corresponding integral equation.

Reviewer: Horst Behnke (Osnabrück)

MSC:

81U05	2-body potential quantum scattering theory
34L25	Scattering theory, inverse scattering involving ordinary differential operators
35P25	Scattering theory for PDEs
35Q40	PDEs in connection with quantum mechanics

Keywords:

jost functions; low energy estimates

Full Text: DOI Link