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Aggregation, blowup, and collapse: the ABC's of taxis in reinforced random walks. (English)

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Summary: In many biological systems, movement of an organism occurs in response to a diffusible or otherwise transported signal, and in its simplest form this can be modeled by diffusion equations with advection terms of the form first derived by *C. S. Patlak* [Bull. Math. Biophys. 15, 311-338 (1953)]. However, other systems are more accurately modeled by random walkers that deposit a nondiffusible signal that modifies the local environment for succeeding passages. In these systems, one example of which is the myxobacteria, the question arises as to whether aggregation is possible under suitable hypotheses on the transition rules and the production of a control species that modulates the transition rates. *B. Davis* [Probab. Theory Relat. Fields 84, 203-229 (1990; Zbl 0665.60077)] has studied this question for a certain class of random walks, and here we extend this analysis to the continuum limit of such walks.

We first derive several general classes of partial differential equations that depend on how the movement rules are affected by the local modulator concentration. We then show that a variety of dynamics is possible, which we classify as aggregation, blowup, or collapse, depending on whether the dynamics admit stable bounded peaks, whether solutions blow up in finite time, or whether a suitable spatial norm of the density function is asymptotically less than its initial value.

MSC:

35Q80	Applications of PDE in areas other than physics (MSC2000)
60J99	Markov processes
92B05	General biology and biomathematics

Keywords:

aggregation; blowup; collapse; chemotaxis equations; diffusion approximation; reinforced random walk

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