Abel, R. J. R.; Bennett, F. E.; Ge, G.
Resolvable perfect Mendelsohn designs with block size five. (English) [Zbl 0991.05012]

A \((v, k, \lambda)\)-Mendelsohn design (MD) is, using graph-theoretic notation, a decomposition of the complete
directed multigraph \(\lambda D_{v}^k\) on \(v\) vertices into \(k\)-circuits. A perfect Mendelsohn design has the additional
property that for any \(r, 1 \leq r \leq k - 1\), and for any two distinct vertices \(x\) and \(y\), there are exactly \(\lambda\)
of the \(k\)-circuits along which the (directed) distance from \(x\) to \(y\) is \(r\). A Mendelsohn design is said to be
resolvable if the \(k\)-circuits can be partitioned into sets of \(k\)-circuits that in turn partition the vertex set. A
resolvable perfect \((v, k, \lambda)\)-MD is called a \((v, k, \lambda)\)-RPMD. It is shown that the necessary condition for the
existence of a \((v, 5, 1)\)-RPMD, namely that \(v\) is divisible by 5, is sufficient except for \(v \in \{5, 10\}\) and with
the 17 cases \(v \in \{20, 30, 90, 95, 100, 110, 115, 120, 130, 135, 140, 150, 160, 170, 190, 195, 210\}\) unresolved.

Reviewer: Patric R.J. Östergård (Helsinki)

MSC:
05B05 Combinatorial aspects of block designs
05C20 Directed graphs (digraphs), tournaments

Keywords:
perfect Mendelsohn design; resolvable design

Full Text: DOI