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A note on integral bases of unramified cyclic extensions of prime degree. II. (English)

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Manuscr. Math. 104, No. 2, 201-210 (2001).

This note is concerned with a particular problem in Galois module structure, to wit: Suppose L/K is a Kummer extension of prime degree p . One knows that existence of a normal integral basis (NIB) implies existence of a power integral basis (PIB), and the question is whether the converse is true. The main result says that the converse fails systematically, that is, there are infinitely many K admitting a counterexample L , and this for every fixed degree N of K over the rationals that is divisible by $p(p-1)$. The basic method is to exhibit K containing ζ_p and $\alpha \in E_K K^{*p}/K^{*p}$ such that α is singular primary but not congruent to 1 modulo $(\zeta_p - 1)^p$. If one looks for α in the “minus part” of $E_K K^{*p}/K^{*p}$, then one can even make do with $\alpha = \zeta_p$, which suffices for the main result. The authors go on to show (and this is technically more difficult) that in some cases one may also find α as above in the plus part.

This note lends further support to the general idea that p -Hilbert-Speiser fields K (that is: fields over which every tame abelian extension of degree p has NIB) are comparatively rare; see work of *D. R. Replegle*, *K. Rubin*, *A. Srivastav* and the reviewer [J. Number Theory 79, 164–173 (1999; Zbl 0941.11044)] and of Carter (forthcoming).

Reviewer: [Cornelius Greither \(Neubiberg\)](#)

MSC:

11R33 Integral representations related to algebraic numbers; Galois module structure of rings of integers

Cited in **5** Documents

11R23 Iwasawa theory

Keywords:

power integral basis; normal integral basis; Kummer extension; Galois module structure

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