Budimir, I.; Cerone, P.; Pečarić, J.
Inequalities related to the Chebyshev functional involving integrals over different intervals.

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The generalized Chebyshev functional is defined by

\[ T(f, g; a, b, c, d) = M(fg; a, b) + M(fg; c, d) - M(f; a, b)M(g; c, d) - M(f; c, d)M(g; a, b), \]

where \( M(f; a, b) \) is the integral mean of \( f \) on \([a, b] \). Let us suppose that \( f, g : I \to \mathbb{R} \) are measurable on \( I \) and \([a, b], [c, d] \subset I \). Further suppose that \( |f(x) - f(y)| \leq H_1|x - y|^r \) and \( |g(x) - g(y)| \leq H_2|x - y|^s \) for all \( x \in [a, b], y \in [c, d] \) and certain \( H_1, H_2 > 0; r, s \in (0, 1] \). If the integrals involved exist, then one has

\[
(k + 1)(k + 2)|T(f, g; a, b, c, d)| \\
\leq \frac{(H_1H_2)}{(b - a)(d - c)}(b - c)^{k+2} - (b - d)^{k+2} + (d - a)^{k+2} - (c - a)^{k+2},
\]

where \( k = r + s \). The Lipschitzian case is also considered. Other results involve a weighted generalized Chebyshev functional.

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MSC:

26D15 Inequalities for sums, series and integrals

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Grüss inequality; Chebyshev inequality; Lupaş inequality; Hölder inequality; generalized integral inequalities; Chebyshev functional

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