Let $M$ be a three-manifold and $K_t(M)$ be its Kauffman bracket skein module, that is, the $\mathbb{C}[t, t^{-1}]$-module generated by the isotopy classes of framed links in $M$ modulo the relations of the Kauffman bracket.

If $K$ is a knot in the three-sphere and $M$ is its complement, then $K_t(T^2 \times I)$ acts from the left on $K_t(M)$ with $T^2$ a torus. The peripheral ideal $I_t(K)$ is defined to be the left ideal of $K_t(T^2 \times I)$ annihilating the empty link $\emptyset$ in $M$. The $A$-ideal, which is shown to be a generalization of the $A$-polynomial, can be defined by $I_t(K)$ [C. Frohman, R. Gelca, and W. LoFaro, Trans. Am. Math. Soc. 354, No. 2, 735-747 (2002; Zbl 0980.57002)]. Here the $A$-polynomial is a two-variable polynomial invariant of a knot defined by using the character variety of $SL(2; \mathbb{C})$-representations of $\pi_1(M)$ [D. Cooper, M. Culler, H. Gillet, D. D. Long and P. B. Shalen, Invent. Math. 118, No. 1, 47-84 (1994; Zbl 0842.57013)].

A pairing $K_t(D^2 \times S^1) \times K_t(M) \to \mathbb{C}[t, t^{-1}]$ is defined by glueing the solid torus $D^2 \times S^1$ to the knot complement $M$. Let $S_n(\alpha)$ be the skein obtained as a Chebyshev polynomial of $\alpha = \{0\} \times S^1 \subset D^2 \times S^1$. Then $(S_n(\alpha), \emptyset)$ defines the $n$th colored Kauffman bracket, a version of the colored Jones polynomial.

Using these facts the author proves that for a knot $K$ and a nonzero element $\psi \in I_t(K)$ there exists a number $\nu$ such that the first $\nu$ colored Kauffman brackets of $K$ and $\psi$ determine all the other colored Kauffman brackets. He also gives a technical condition that the $A$-ideal of a knot determines all the Kauffman brackets. As an example a recursive formula for the colored Kauffman brackets of the trefoil knot is given.

Reviewer: Hitoshi Murakami (Tokyo)

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