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Consistently there is no non trivial ccc forcing notion with the Sacks or Laver property.
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Summary: At a recent set theory conference, Boban Veličković asked the following question:

Question 1.1. Is there a nontrivial forcing notion with the Sacks property which is also ccc?

A “definable” variant of this question has been answered by the author [Isr. J. Math. 88, 159-174 (1994; Zbl 0815.03031)]:

Every nontrivial Souslin forcing notion which has the Sacks property has an uncountable antichain.

We show here: Theorem 1.2. The following statement is equiconsistent with ZFC:

(*) Every nontrivial forcing notion which has the Sacks property has an uncountable antichain.

Independently, Veličković has also proved the consistency of (*), following the ideas of the above cited paper and some of his own works. In fact, he shows that the proper forcing axiom (PFA), and even the open coloring axiom, imply (*).

Our proof shows that also the following strengthening of (*) is equiconsistent with ZFC:

(**) Every nontrivial forcing notion which has the Laver property has an uncountable antichain.

So our result and Veličković’s result are incomparable.

MSC:

03E05 Other combinatorial set theory

03E35 Consistency and independence results

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