

**Garrett, Paul****Euler factorization of global integrals.** (English) [Zbl 1002.11042](#)

Doran, Robert S. (ed.) et al., Automorphic forms, automorphic representations, and arithmetic. Proceedings of the NSF-CBMS regional conference in mathematics on Euler products and Eisenstein series, Fort Worth, TX, USA, May 20-24, 1996. Dedicated to Goro Shimura. Providence, RI: American Mathematical Society. Proc. Symp. Pure Math. 66(pt.2), 35-101 (1999).

Summary: We consider integrals of cusp forms  $f$  on reductive groups  $G$  defined over number fields  $k$  against restrictions  $\iota^*E$  of Eisenstein series  $E$  on “larger” reductive groups  $\tilde{G}$  over  $k$  via imbeddings  $\iota : G \rightarrow \tilde{G}$ . We give hypotheses sufficient to assure that such global integrals have Euler products. At good primes, the local factors are shown to be rational functions in the corresponding parameters  $q^{-s}$  from the Eisenstein series and in the Satake parameters  $q^{-s_i}$  coming from the spherical representations locally generated by the cusp form. The denominators of the Euler factors at good primes are estimated in terms of “anomalous” intertwining operators, computable via orbit filtrations on test functions. The standard intertwining operators (attached to elements of the (spherical) Weyl group) among these unramified principal series yield symmetries among the anomalous intertwining operators, thereby both sharpening the orbit-filtration estimate on the denominator and implying corresponding symmetry in it.

Finally, we note a very simple dimension-counting heuristic for fulfillment of our hypotheses, thereby giving a simple test to exclude configurations  $\iota : G \rightarrow \tilde{G}$ ,  $E$ , as candidates for Euler product factorization. Some simple examples illustrate the application of these ideas.

For the entire collection see [[Zbl 0919.00046](#)].

**MSC:**

- [11F66](#) Langlands  $L$ -functions; one variable Dirichlet series and functional equations
- [11F11](#) Holomorphic modular forms of integral weight
- [22D30](#) Induced representations for locally compact groups

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**Keywords:**

[automorphic  \$L\$ -function](#); [unramified principal series](#); [Rankin-Selberg integral](#); [periods](#); [integrals of cusp forms](#); [reductive groups](#); [Eisenstein series](#); [global integrals](#); [local factors](#); [spherical representations](#); [denominators of the Euler factors at good primes](#); [intertwining operators](#); [Euler product factorization](#)